

# Experimental restraints on molecular shape for protein structure calculations

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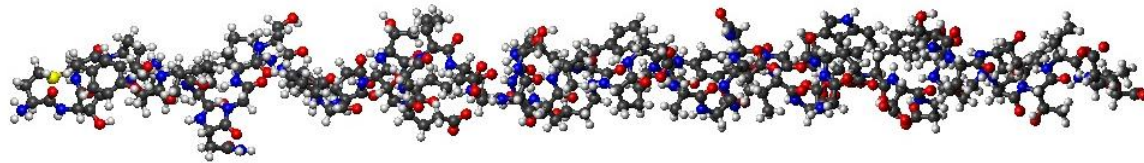


## OUTLINE

- Global and local restraints for protein structure calculations
- Global restraints on overall shape from NMR relaxation data
- Combination of global and local restraints in NMR relaxation data
- NMR relaxation data as the source of dynamic information about protein domain motions

# Ultimate Goal of protein structure prediction

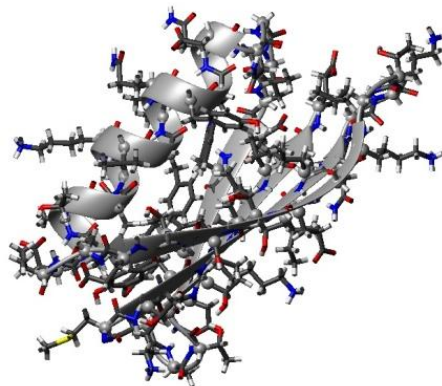
Sequence of amino acid residues



+

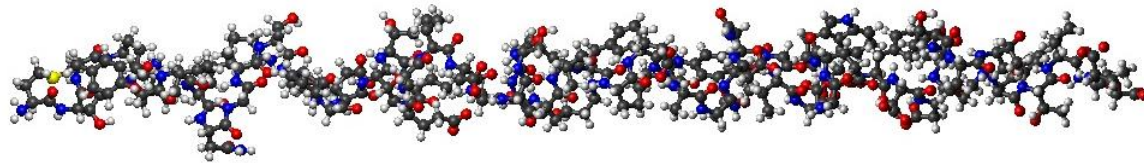
Model inter atomic forces

=

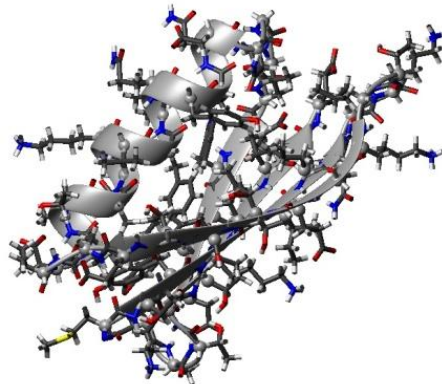


# Ultimate Goal of protein structure prediction

Sequence of amino acid residues



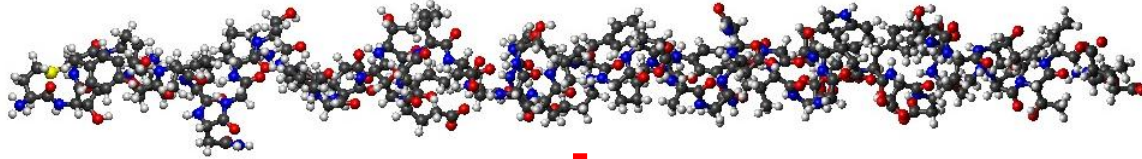
Model inter atomic forces



**Is not yet  
accomplished**

# NMR Protein structure determination

Sequence of amino acid residues



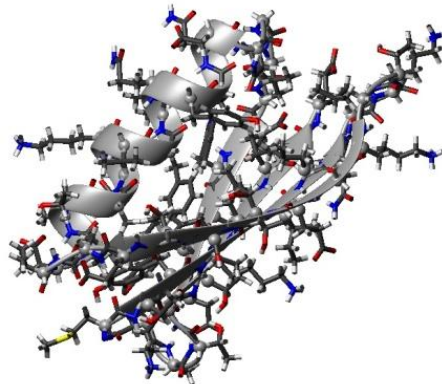
+

Experimental restraints: NOE, RDC, SAXS and etc.

+

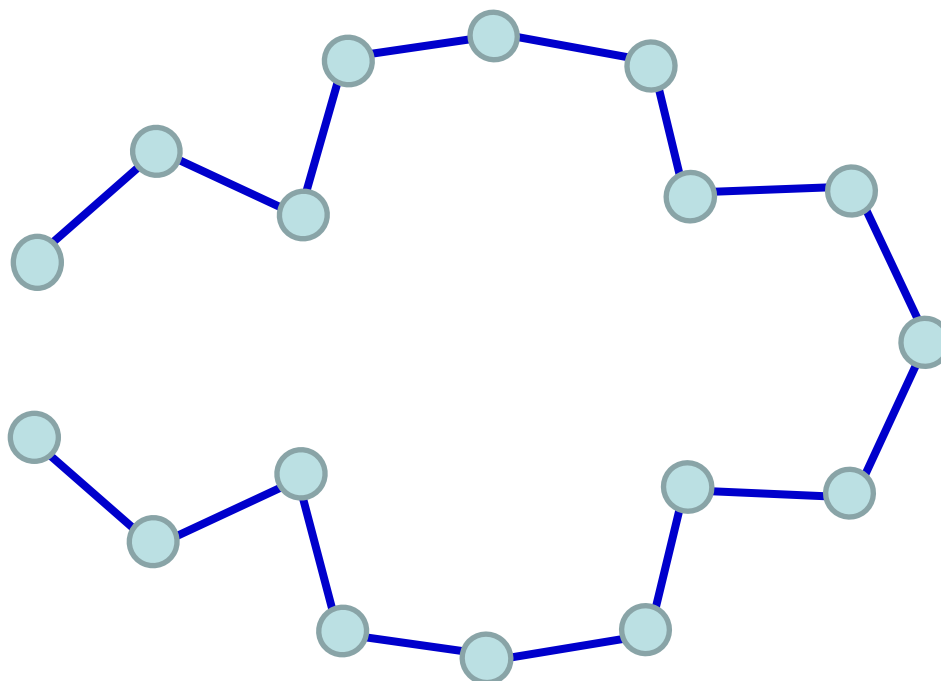
*a priori* restraints from database of known structures

=



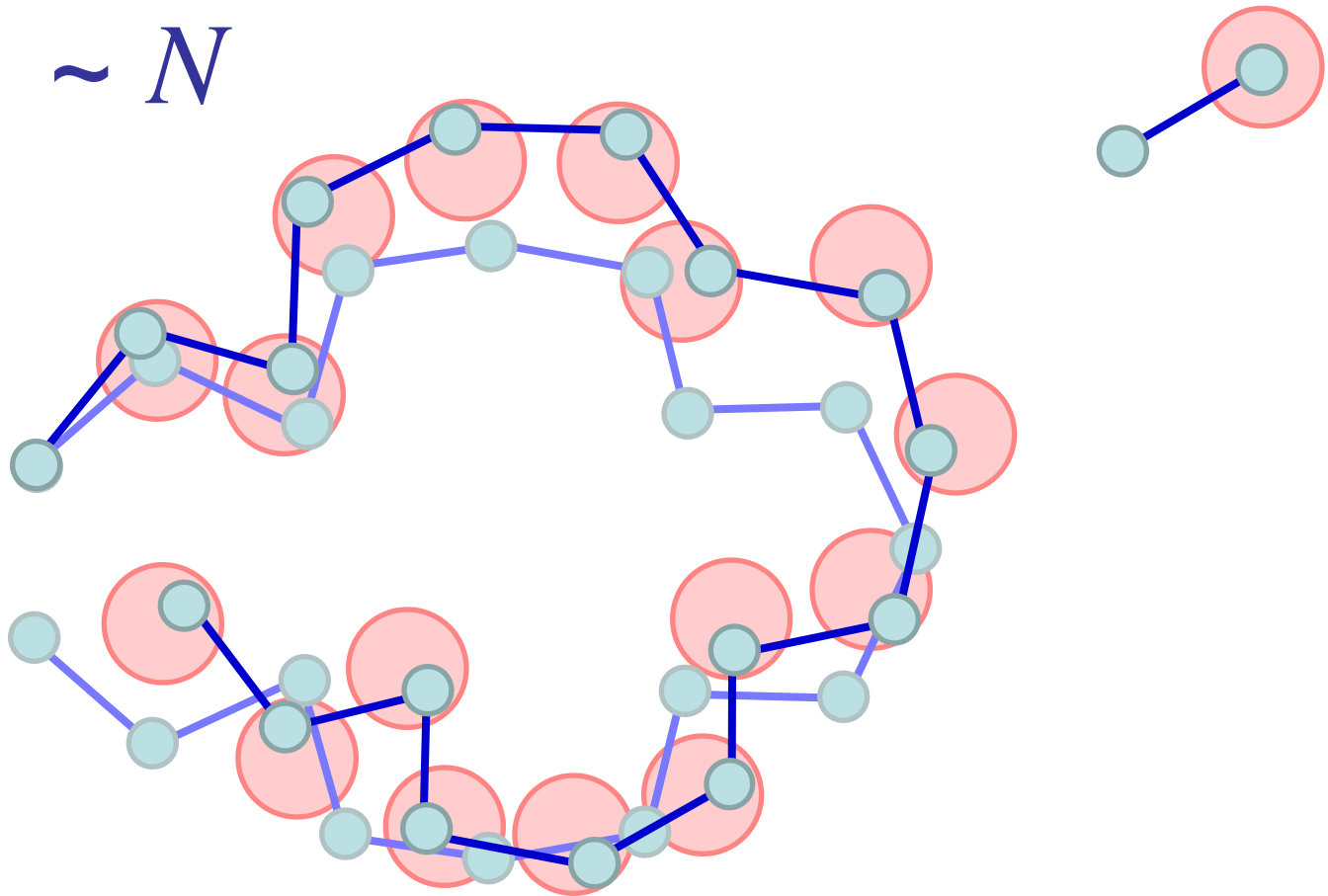
## Local and Global restraints

Ideal structure



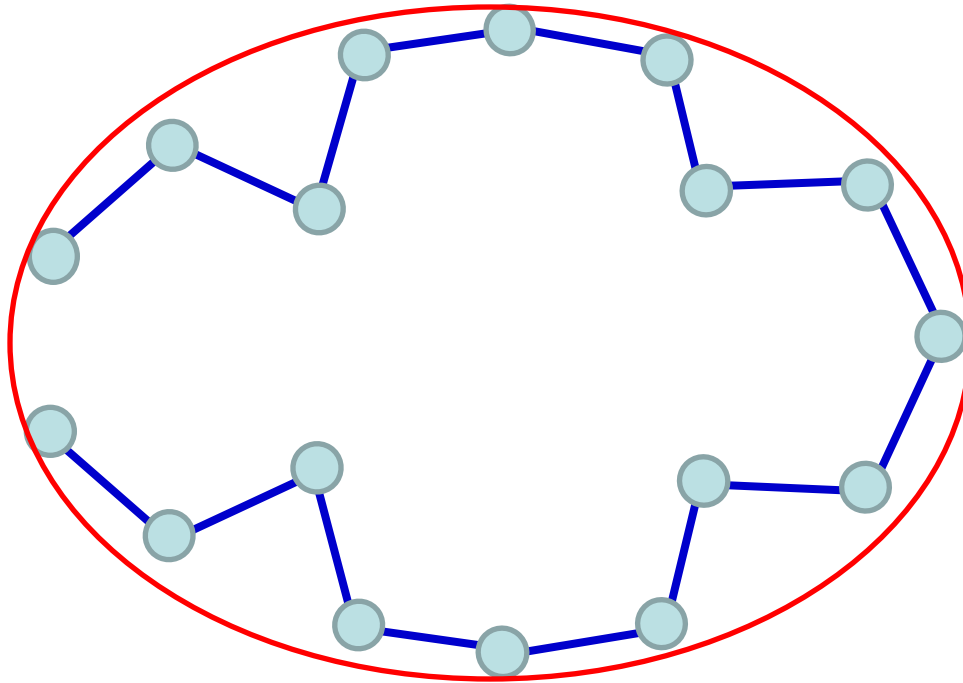
## Local restraints

$$\langle \Delta^2 \rangle \sim N$$



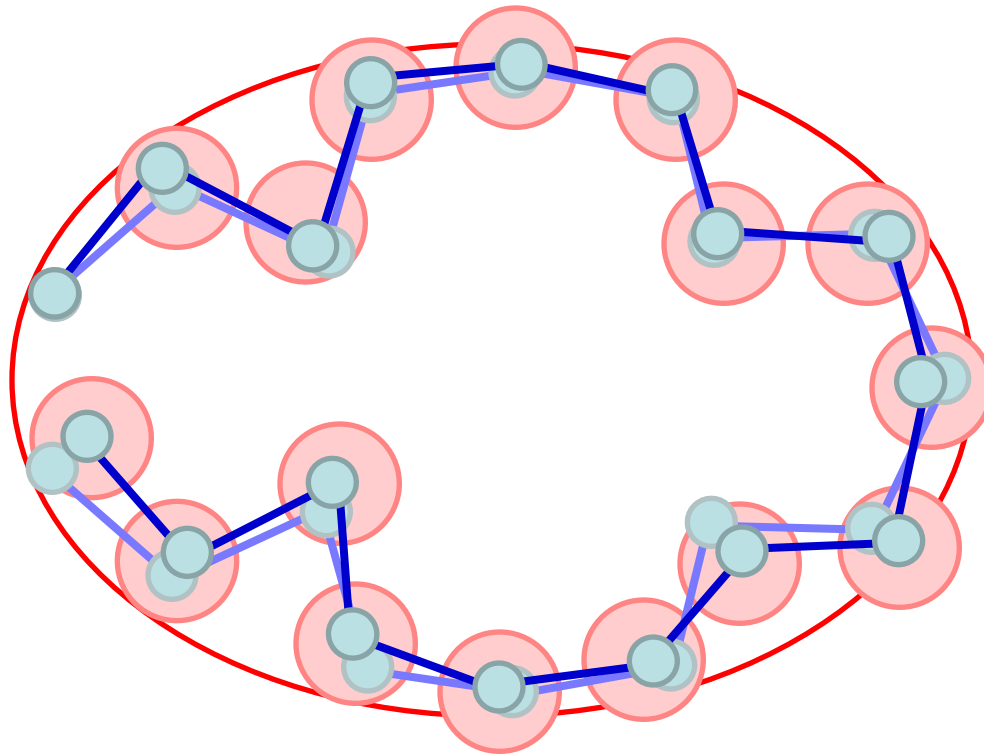
## Global restraints

Overall shape





## Local and Global restraints



## Some possible experimental sources of global restraints

- **Radius of gyration**

could be obtained from empiric relationship between protein size and radius of gyration

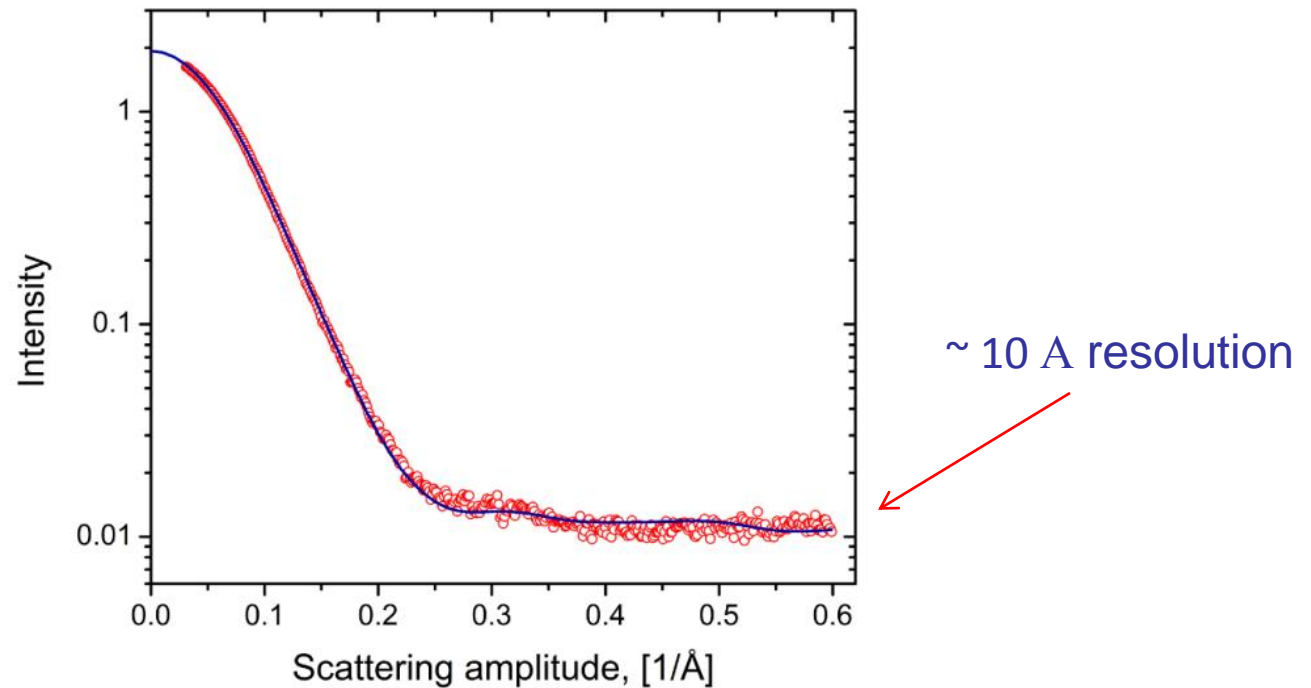
$$R_g = 2.2 N^{0.38} \text{ \AA}$$

or directly from SAXS curve at the limit at the of small scattering angles (Guinier limit)

$$\ln I(Q) = \ln I(0) - \frac{1}{3} R_g^2 Q^2$$

## Some possible experimental sources of global restraints

- Radius of gyration
- **SAXS curve**



## Some possible experimental sources of global restraints

- Radius of gyration
- SAXS curve
- NMR relaxation data

## Sampling protein surface

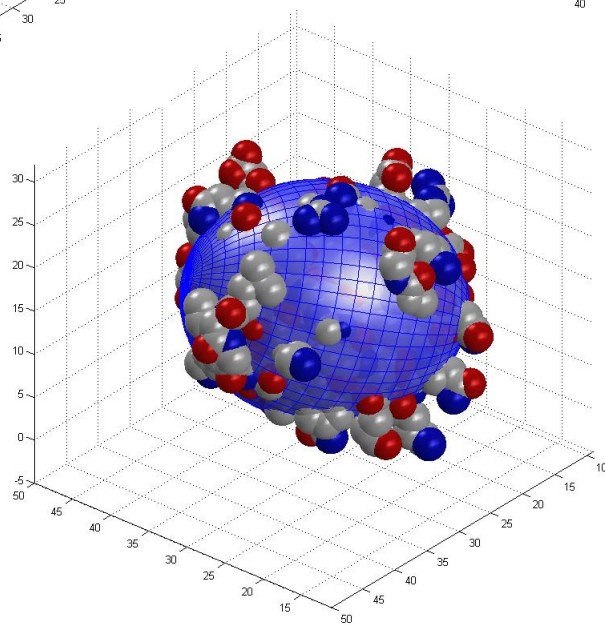
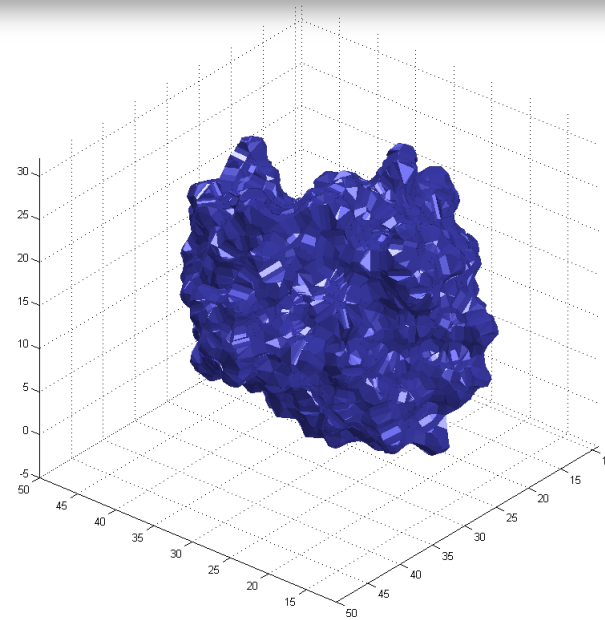
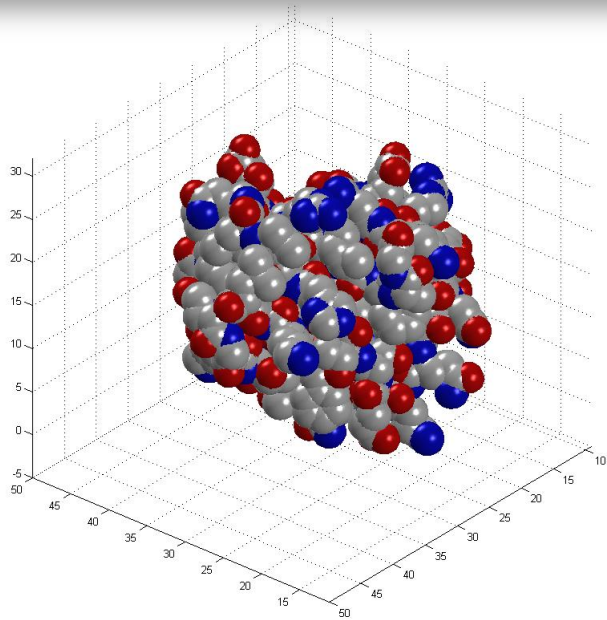
Implemented into specific Xplor NIH module and already used for

- Evaluation protein *Rotation Diffusion* properties
- Evaluation of *NMR relaxation* rates
- Simulation *SAXS scattering curve*

and **potentially can be used for evaluation of**

- *Translational Diffusion* coefficients
- *Residual Dipolar Coupling* originated from steric hindrances
- etc.

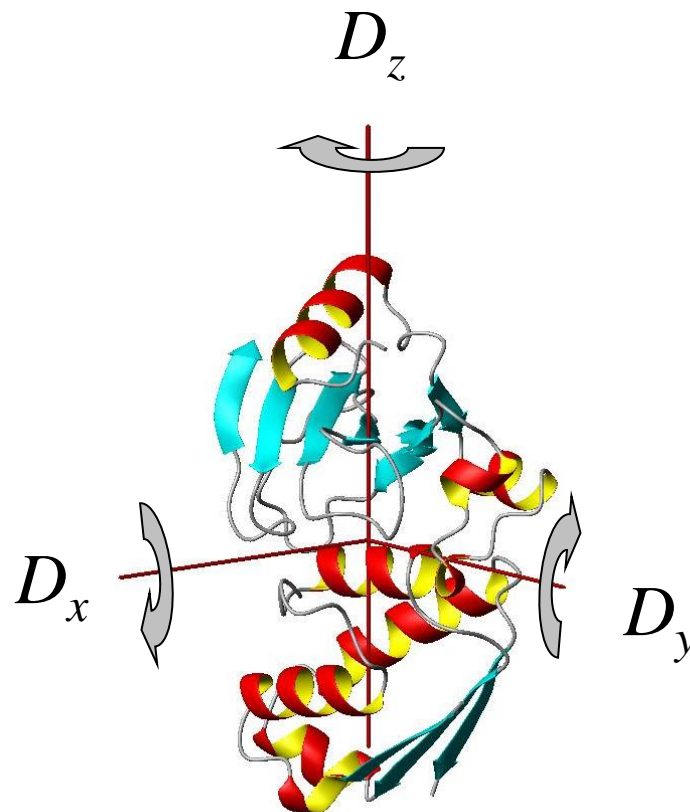
# Sampling protein surface



## Overall shape restraints from Diffusion tensor

Diffusion Tensor

$$\begin{bmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{bmatrix}$$



**3 Euler angles for  
Diffusion Tensor PAF**

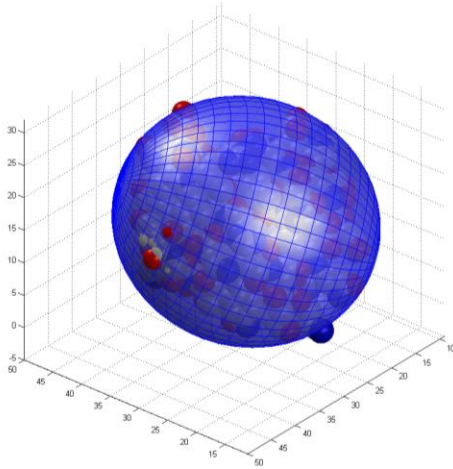
## Effect of overall protein shape on NMR relaxation data

Rotation Diffusion tensor

$$\begin{matrix} R_2 \\ R_1 \end{matrix} \leftarrow J(\omega) \leftarrow C(t) \sim \sum_{n,p,q} e^{-E_n t} A_{np} A_{nq} Y_2^p(\Omega) Y_2^{q*}(\Omega)$$



# Schematic Procedure



I) Build equivalent ellipsoid

II) Evaluate components of diffusion tensor

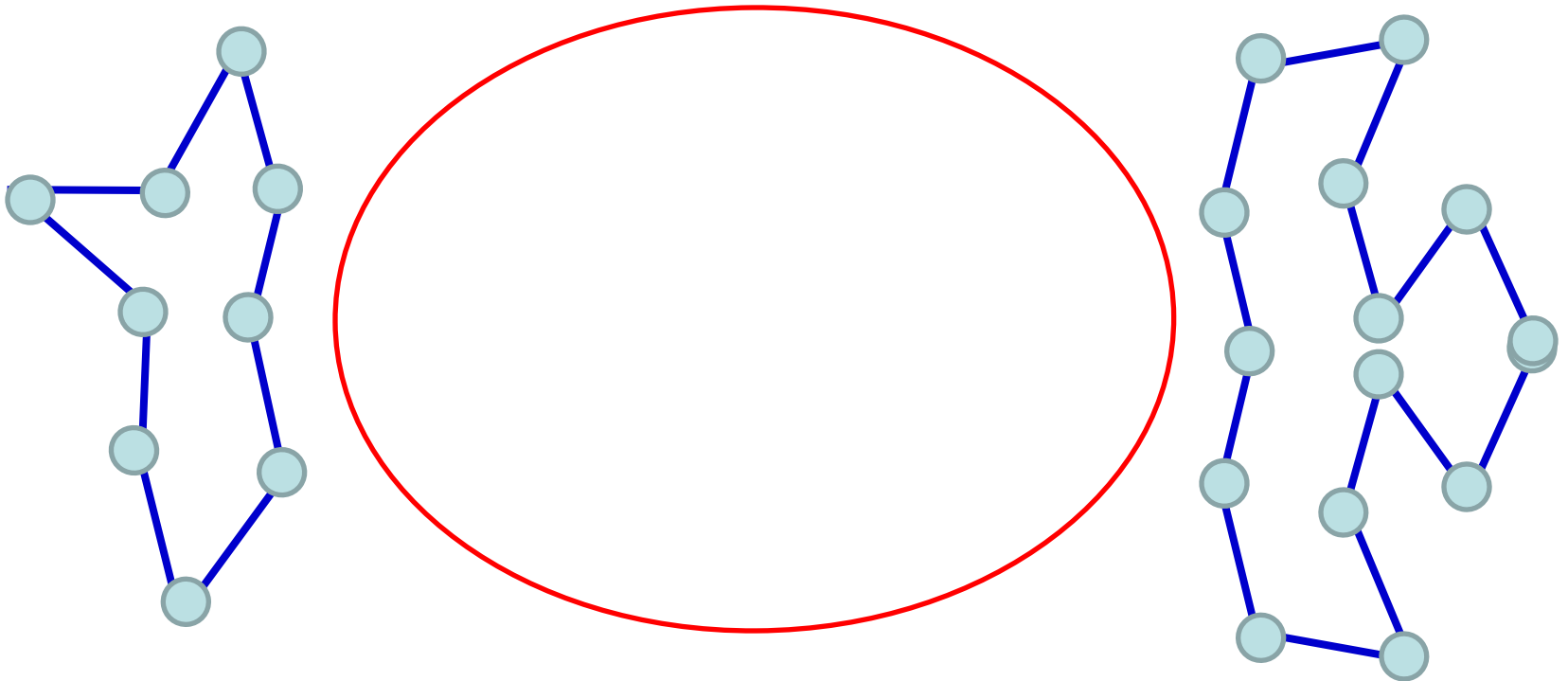
$$\begin{bmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{bmatrix}$$

III) Compare evaluated components of Diffusion tensor with experimental values obtained from NMR

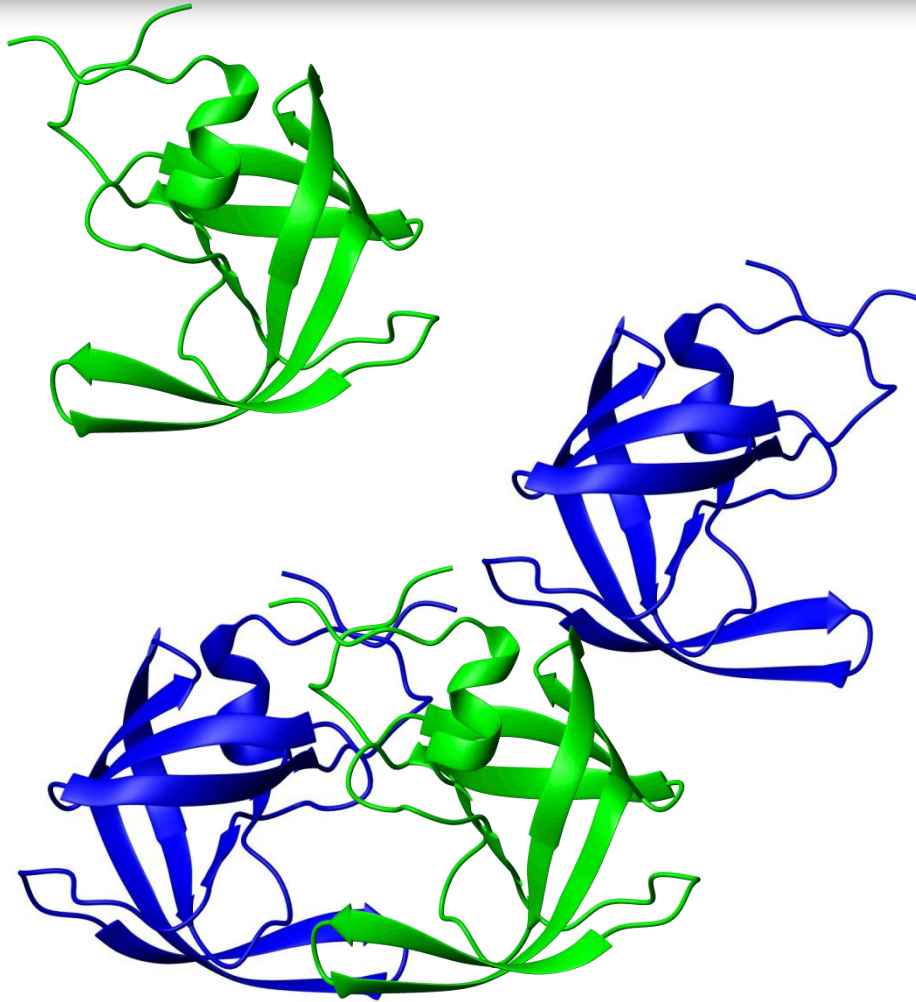
$$\chi^2 = \sum_{\substack{i=1,3 \\ j=i,3}} \left( D_{i,j}^{calc} - D_{i,j}^{exp} \right)^2$$

# Assembling structures of multi domain proteins

Global restraints on Overall shape



## Assembling structure of a symmetric protein homo dimer



Generic docking protocol

Part I:

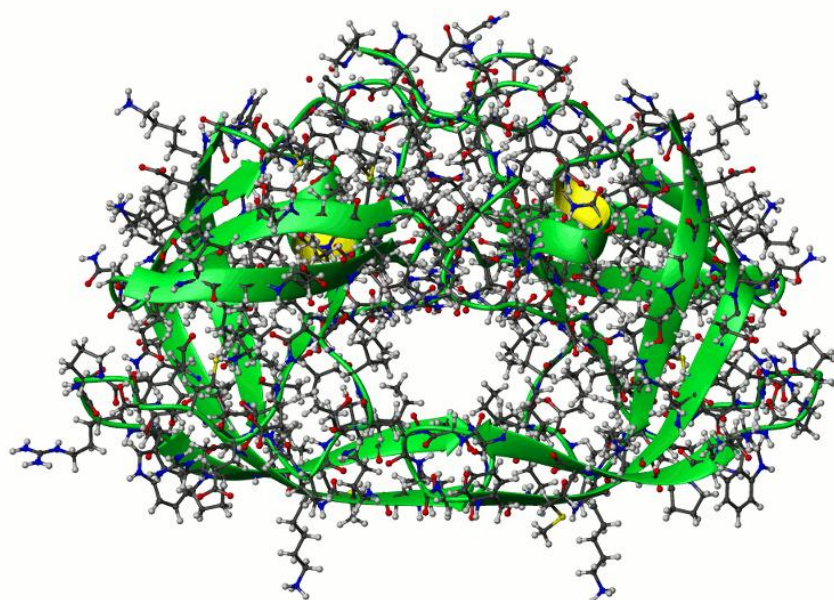
Rigid body dynamics  
for raw domain positioning.

Part II:

Simulated annealing  
with flexible side chains  
for final adjustment.

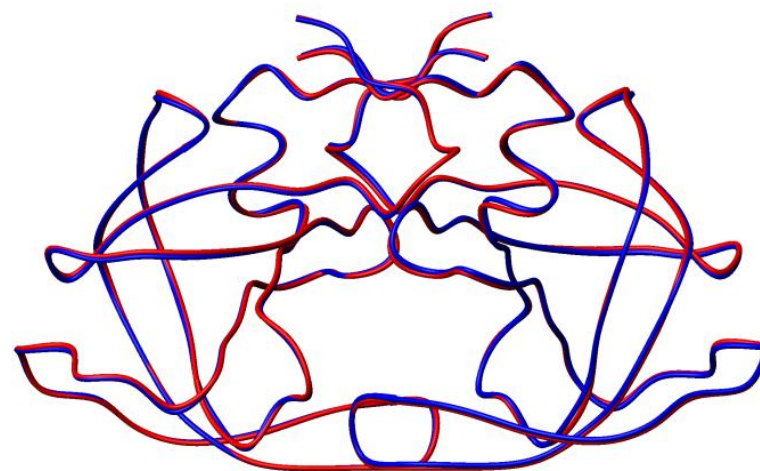
# Assembling structure of a symmetric protein homo dimer

HIV -1 protease



10 lowest energy structures

Shape restraints from  
Components of diffusion tensor

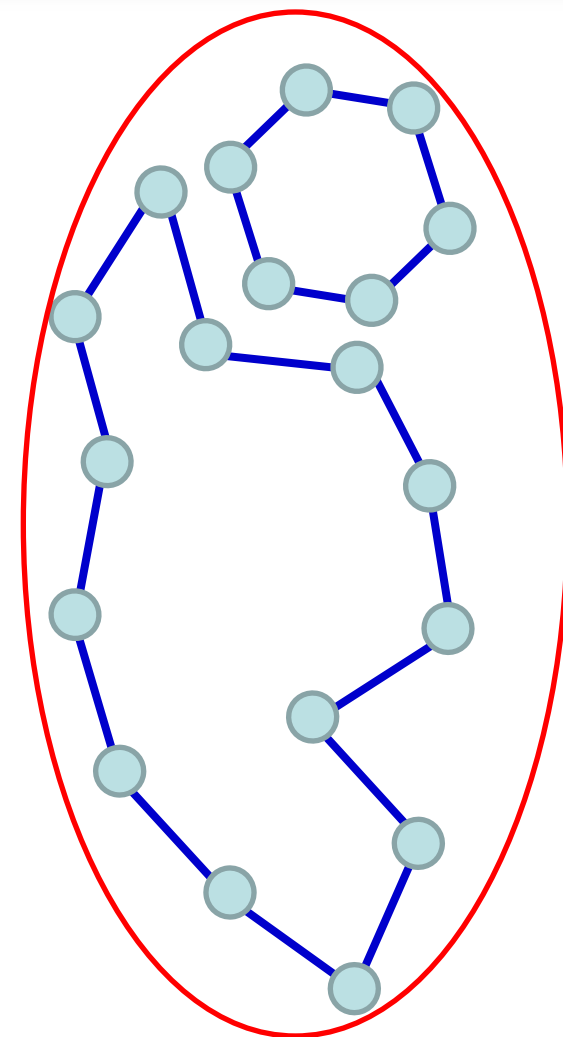
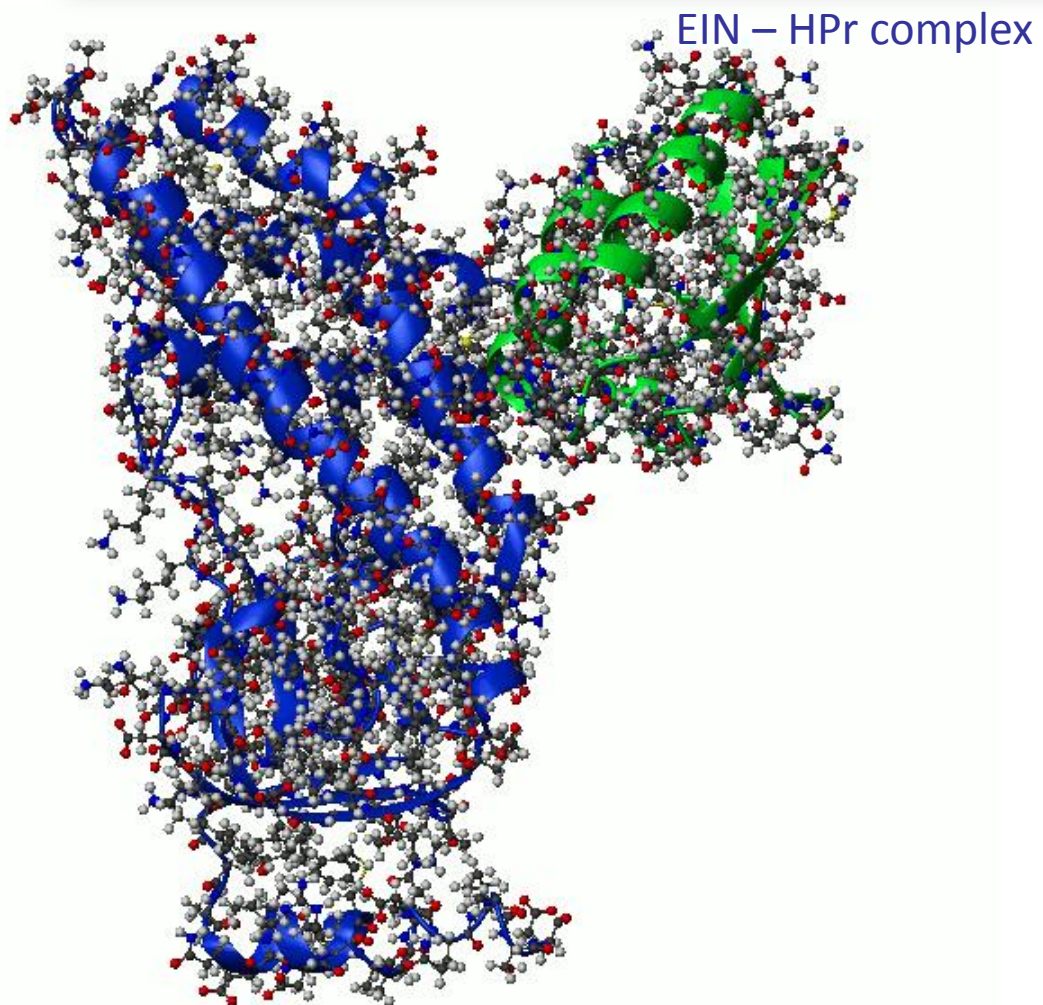


Averaged over 10 lowest energy structures  
(blue) versus reference (red)

$C\alpha$  RMSD

$0.35 \pm 0.09$  [Å]

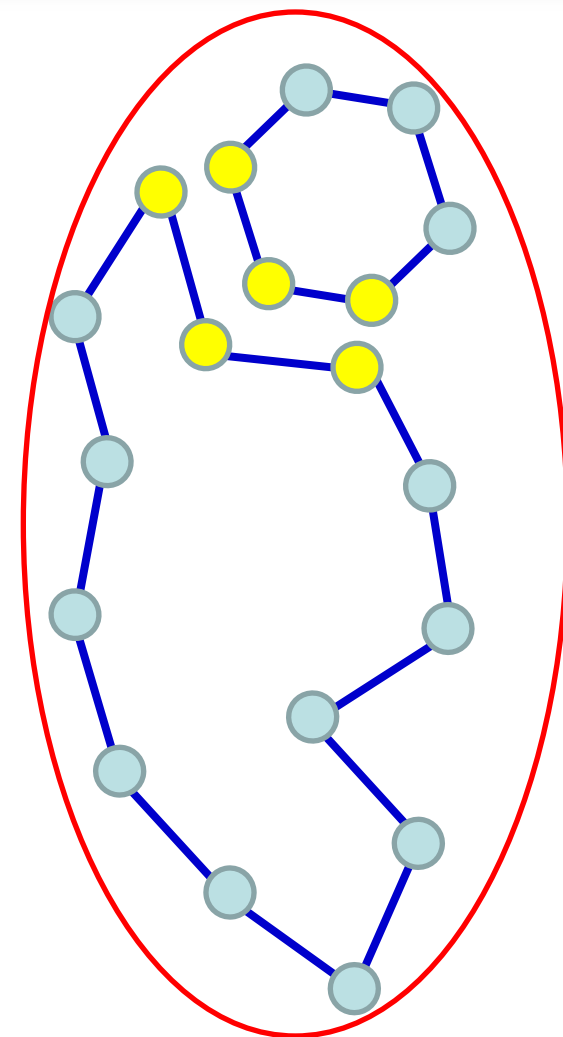
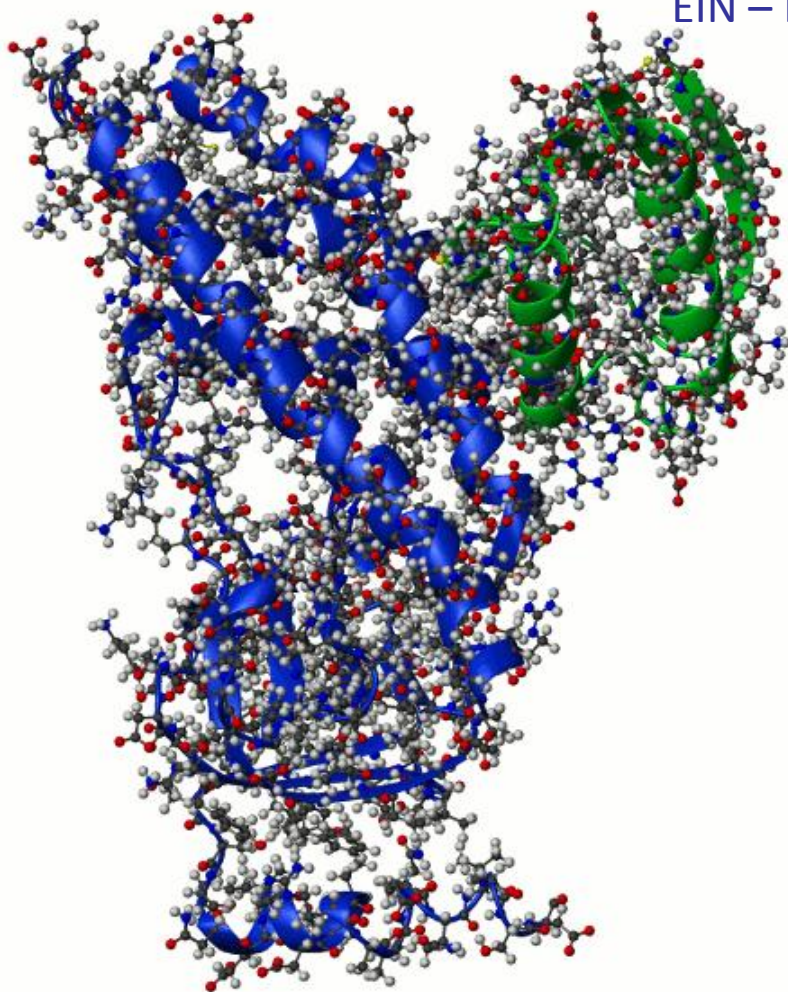
## Application to an asymmetric complex



10 lowest energy structures

## Application to an asymmetric complex

EIN – HPr complex



10 lowest energy structures

## Using both shape and orientation restraints

$$\begin{array}{c}
 R_2 \\
 R_1
 \end{array}
 \leftarrow J(\omega) \leftarrow C(t) \sim \sum_{n,p,q} e^{-E_n t} A_{np} A_{nq} Y_2^p(\Omega) Y_2^{q*}(\Omega)$$

Overall Shape

Bond Orientation

Ratio of relaxation rates “almost” independent of fast local motions

$$\frac{R_2}{R_1} = \frac{4J(0) + 6J(\omega_H - \omega_N) + J(\omega_H + \omega_N) + 6J(\omega_H) + 3J(\omega_N)}{2[6J(\omega_H - \omega_N) + J(\omega_H + \omega_N) + 3J(\omega_N)]}$$

## Using both shape and orientation restraints

$$\begin{array}{c}
 R_2 \\
 R_1
 \end{array}
 \leftarrow J(\omega) \leftarrow C(t) \sim \sum_{n,p,q} e^{-E_n t} A_{np} A_{nq} Y_2^p(\Omega) Y_2^{q*}(\Omega)$$

Overall Shape

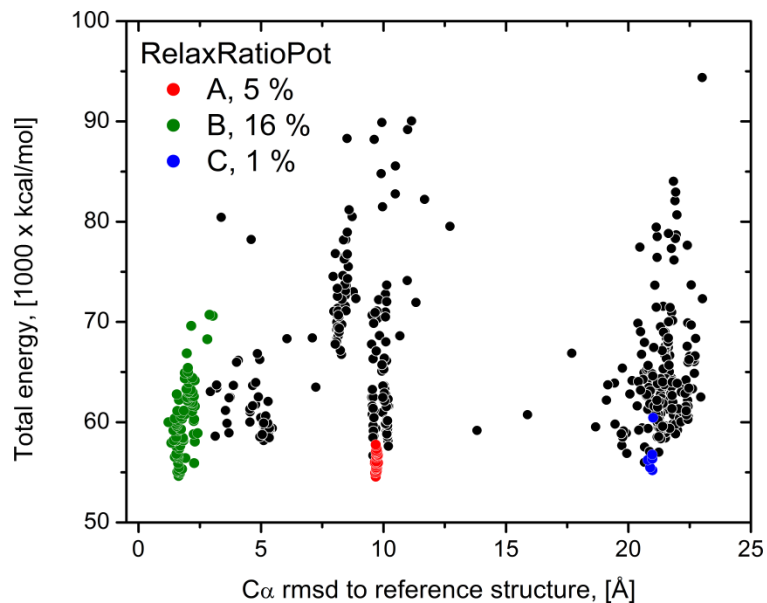
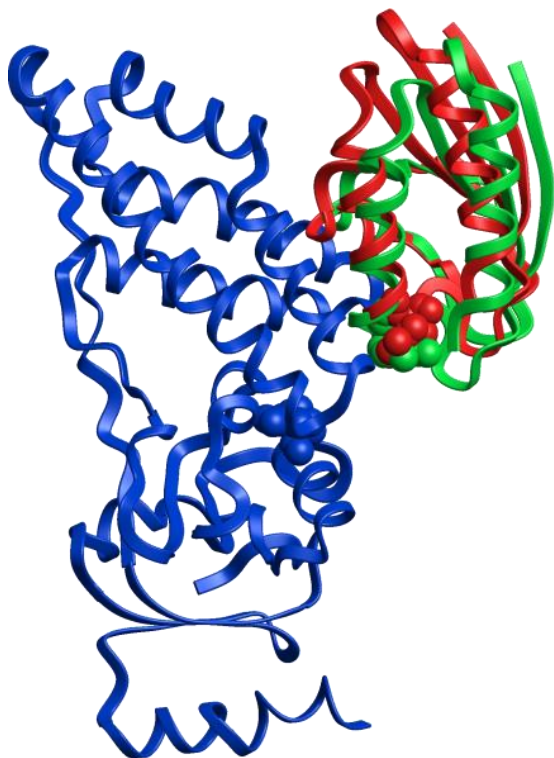
Bond Orientation

$$\chi^2 \sim \sum_i \left( \frac{(R_2/R_1)_i^{calc} - (R_2/R_1)_i^{exp}}{\sigma_i^{err}} \right)^2$$

Energy of a potential term



## Using both shape and orientation restraints

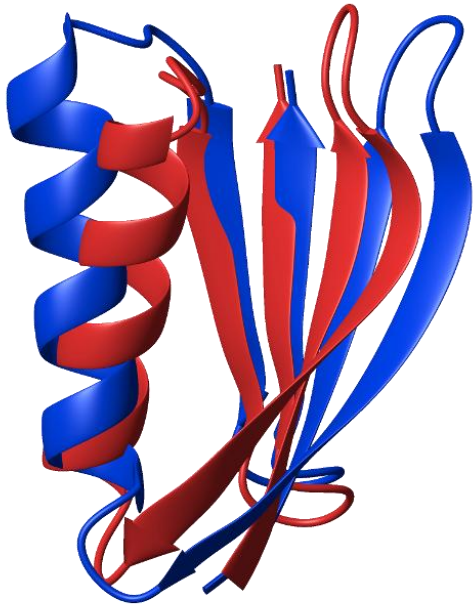


10 lowest energy structures  
from the most populated cluster B

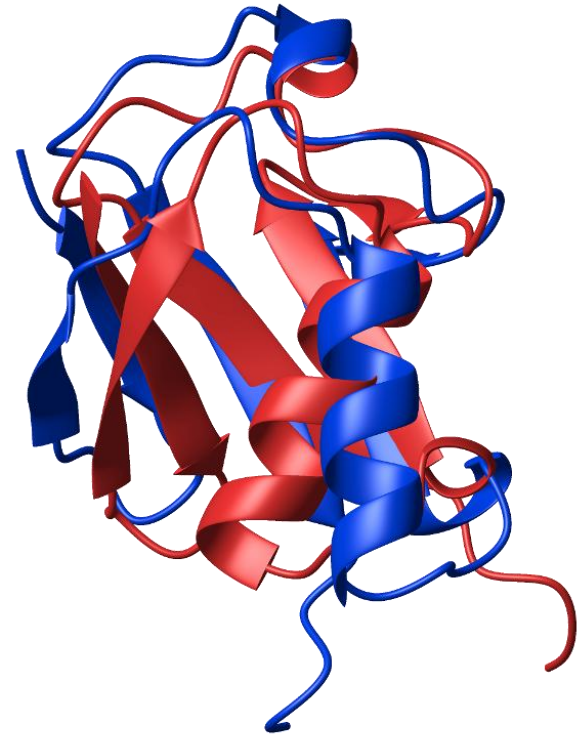
C $\alpha$  RMSD  $1.73 \pm 0.20$  [Å]

## Structure determination of globular proteins

Gb 3: b.b.C $\alpha$  RMSD 3.2 [Å]



Ubiquitin: b.b.C $\alpha$  RMSD 3.5 [Å]



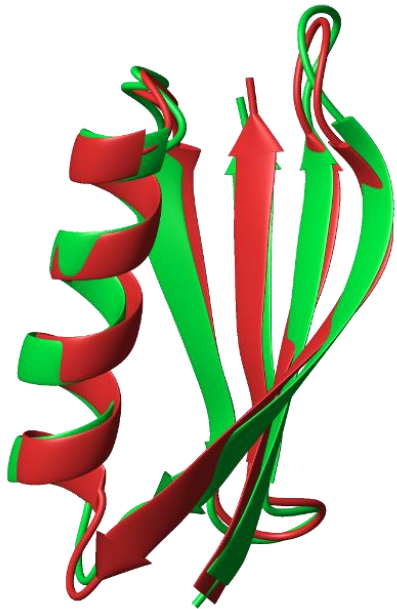
### Experimental restraints:

Dihedral angles from TALOS+ predictions

Back Bone Hydrogen bonds connectivity

# Structure determination of globular proteins

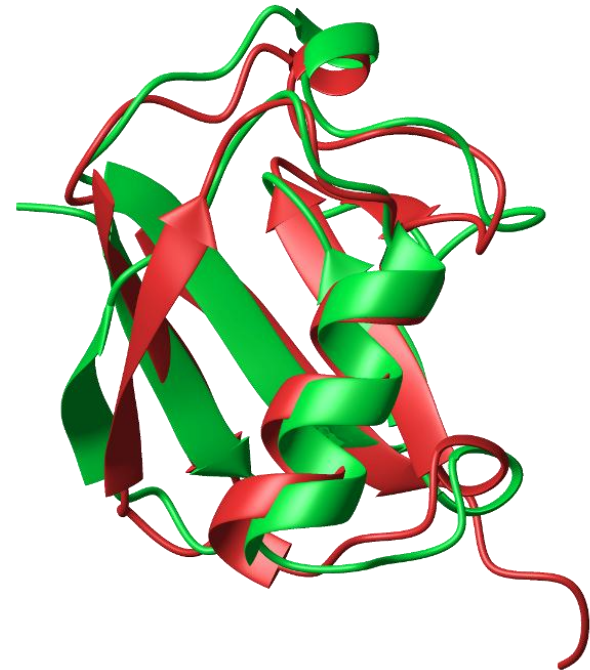
Gb 3: b.b.C $\alpha$  RMSD 1.1 [Å]



## Experimental restraints:

Dihedral angles from TALOS+ predictions  
Back Bone Hydrogen bonds connectivity

Ubiquitin: b.b.C $\alpha$  RMSD 1.8 [Å]

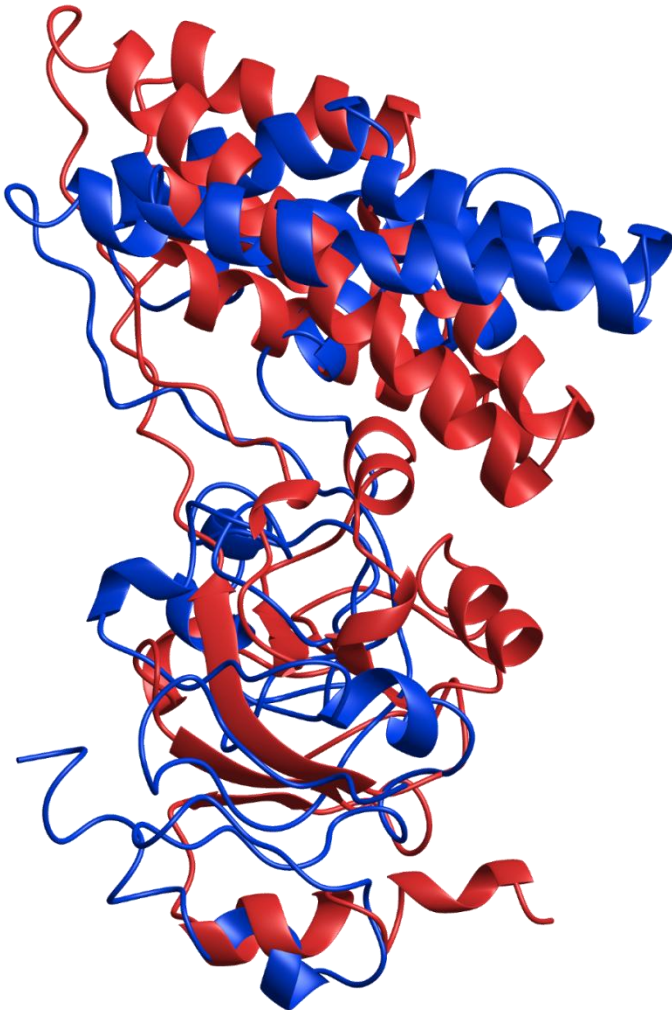


**$R_2/R_1$  ratios**  
+ of  $^{15}\text{N}$  relaxation rates

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# Structure determination of globular proteins

EIN: b.b.C $\alpha$  RMSD 14.7 [Å]



## Experimental restraints:

Dihedral angles from TALOS+ predictions

Back Bone Hydrogen bonds connectivity

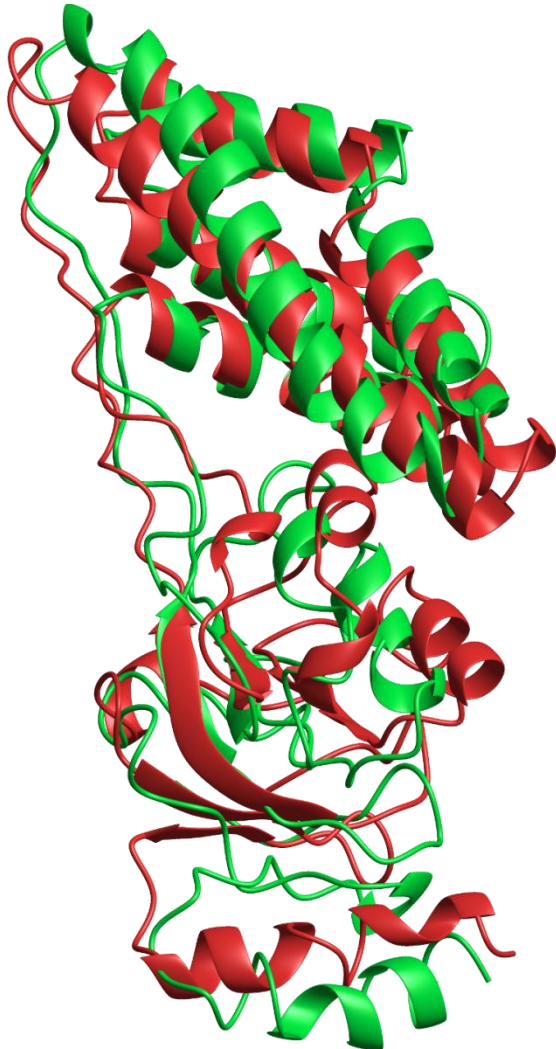
*and*

limited set of 804 NOEs

for methyl and HN protons

# Structure determination of globular proteins

EIN: b.b.C $\alpha$  RMSD 4.1 [Å]



## Experimental restraints:

Dihedral angles from TALOS+ predictions  
Back Bone Hydrogen bonds connectivity

*and*

limited set of 804 NOEs  
for methyl and HN protons

+

**$R_2/R_1$  ratios**  
of  $^{15}\text{N}$  relaxation rates

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in every **Xplor-NIH** installation

~/xplor/eginput/

Example scripts for using DiffPot and RelaxaRatioPot

Sample scripts for

- Docking
- Refinement
- Structure determination

with all necessary sample data files

Helper scripts for data fitting and filtering

## Future challenges

### Experimental / Spectroscopic

- Temperature control and calibration
- Viscosity measurements
- Better spectroscopic techniques

### Computational / Theoretical

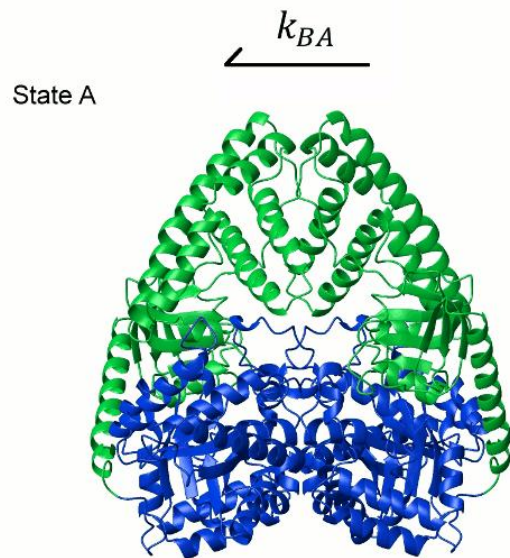
- Inhomogeneous hydration layer
- Treatment of internal motions
- Better models for diffusion tensor predictions

## Theoretical Modeling of internal protein motions

- Local mobility of small protein subunits
- Motions of Large protein domains



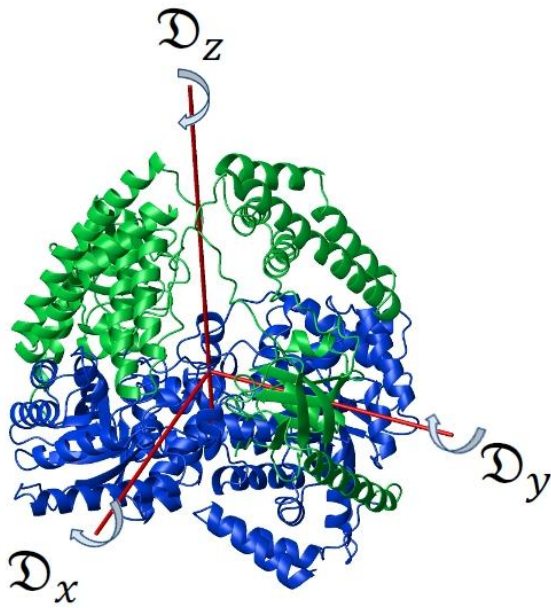
## Time scale of Conformational transitions



$$\tau_c = \frac{1}{k_{AB} + k_{BA}}$$

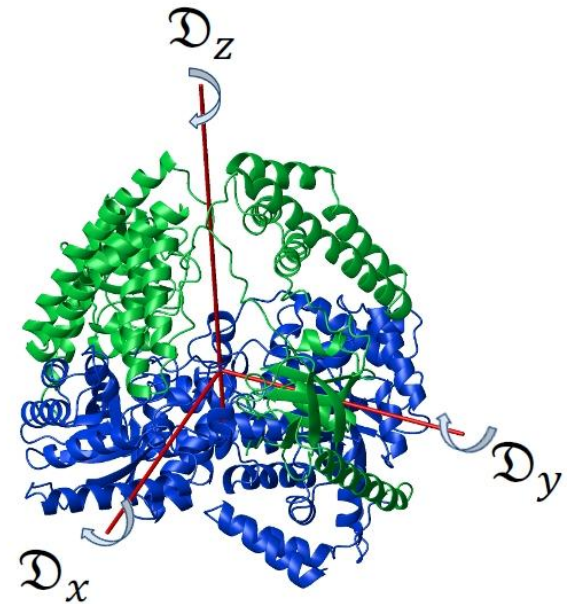
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## Time scale of Rotational diffusion



$$\tau_{\mathcal{D}} = \frac{1}{2(\mathcal{D}_x + \mathcal{D}_y + \mathcal{D}_z)}$$

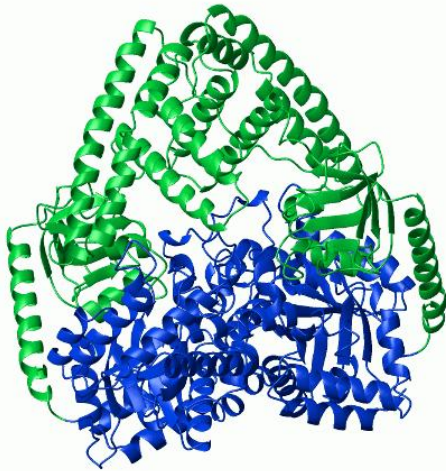
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# Conformation transitions on different time scales

## Slow Exchange

State A



$$\tau_c \gg \tau_D$$

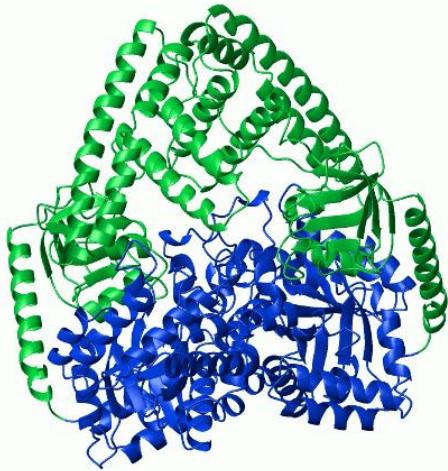
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Approximation of **two species**  
with **two different structures**  
and **two diffusion tensors**

$$\mathcal{D}^A \text{ and } \mathcal{D}^B$$

# Conformation transitions on different time scales

State A



Fast Exchange

$$\tau_c \ll \tau_D$$

---

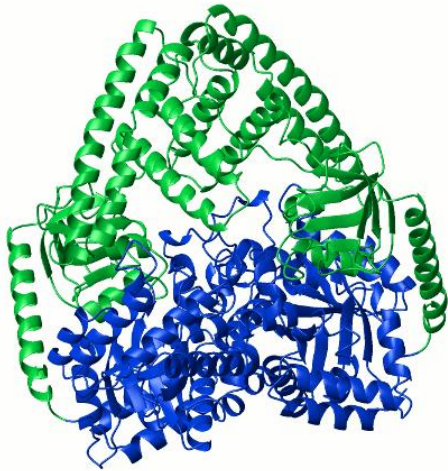
Approximation of **single conformer**  
with **one averaged structure**  
and **one averaged diffusion tensor**

$\bar{D}$

# Conformation transitions on different time scales

## Intermediate Exchange

State A



$$\tau_c \sim \tau_D$$

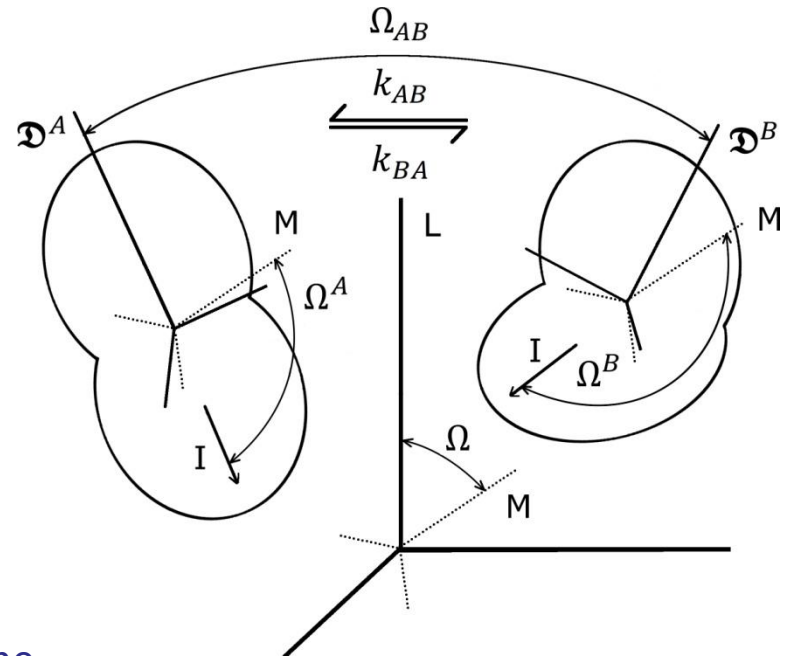
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## Rotational diffusion of semi-Rigid molecule

### Conformation transition between discrete set of states

- Molecule tumbles in isotropic solvent
- Molecule exchanges between discrete conformations  $\varepsilon = A, B, \dots$
- In each conformation state molecule is rigid and have diffusion tensor  $\mathfrak{D}^\varepsilon$
- The transition time is much shorter than the time which molecule spends in any conformation



## Rotational diffusion of semi-Rigid molecule

Correlation function in frequency domain (no restrictions)

$$\tilde{C}_l(\sigma) = \frac{4\pi}{2l+1} \sum_{\varepsilon, \eta} \mathbf{Y}_l^T(\Omega_{\varepsilon l}) \mathbf{A}^{\varepsilon, l\dagger} \mathbf{R}^{\varepsilon \eta}(\sigma) \mathbf{A}^{\eta, l} \mathbf{Y}_l^*(\Omega_{\eta l}) P_{eq}^{\eta}$$

## Rotational diffusion of semi-Rigid molecule

Correlation function in frequency domain (no restrictions)

$$\tilde{C}_l(\sigma) = \frac{4\pi}{2l+1} \sum_{\varepsilon, \eta} \mathbf{Y}_l^T(\Omega_{\varepsilon l}) \mathbf{A}^{\varepsilon, l \dagger} \mathbf{R}^{\varepsilon \eta}(\sigma) \mathbf{A}^{\eta, l} \mathbf{Y}_l^*(\Omega_{\eta l}) P_{eq}^{\eta}$$

Spectral density

$$J_l(\omega) = \text{Re} \left\{ \tilde{C}_l(\sigma) \Big|_{\sigma \rightarrow i\omega} \right\}$$

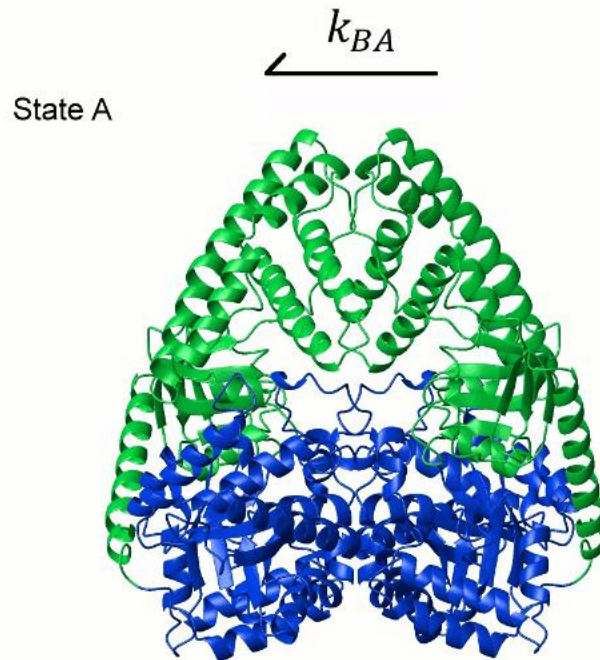
Experimental observables: R1, R2 etc.

Ryabov, Clore, Schwieters (2012)



# Illustrative calculations

EI dimer



Estimations of XplorNIH @ 300 K

$$\mathfrak{D}_x^A = 29.16 \times 10^7 [s^{-1}] \quad \tau_{\mathfrak{D}}^A = 53.73 [ns]$$

$$\mathfrak{D}_y^A = 31.47 \times 10^7 [s^{-1}]$$

$$\mathfrak{D}_z^A = 32.43 \times 10^7 [s^{-1}]$$

$$\mathfrak{D}_x^B = 15.71 \times 10^7 [s^{-1}] \quad \tau_{\mathfrak{D}}^B = 79.99 [ns]$$

$$\mathfrak{D}_y^B = 15.82 \times 10^7 [s^{-1}]$$

$$\mathfrak{D}_z^B = 30.99 \times 10^7 [s^{-1}]$$

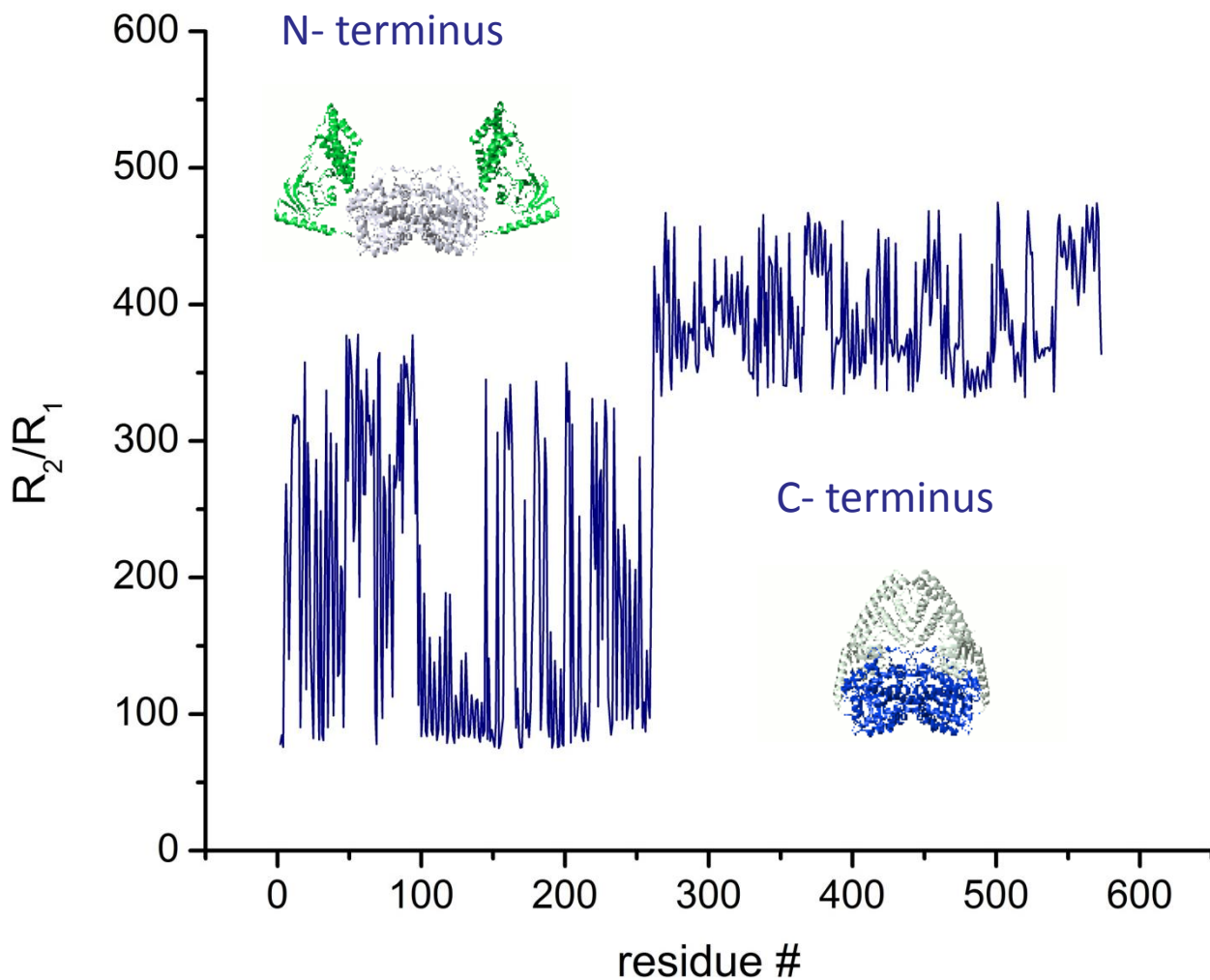
Assumptions

Symmetric motions  $\Omega_{AB}: \{\alpha_{AB} = 0, \beta_{AB} = 0, \gamma_{AB} = 0\}$

Equal occupation  $P_{eq}^A = P_{eq}^B = 1/2 \quad k_{AB} = k_{BA}$

## Illustrative calculations

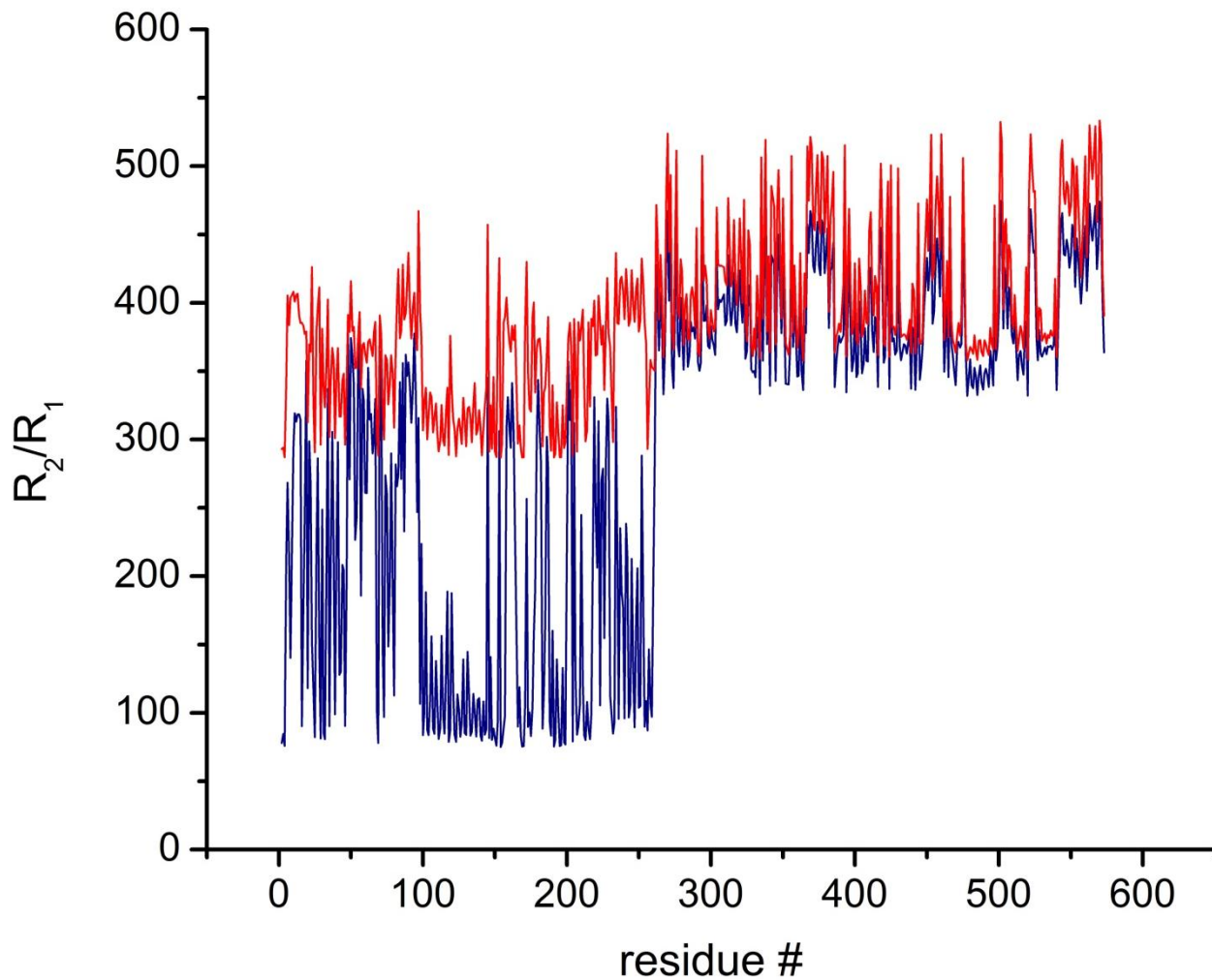
NMR relaxation rates for 600 MHz @ 300 K



$$k = \frac{2}{\tau_D^A + \tau_D^B} = 155.57 \text{ [ns}^{-1}\text{]}$$

## Illustrative calculations

NMR relaxation rates for 600 MHz @ 300 K

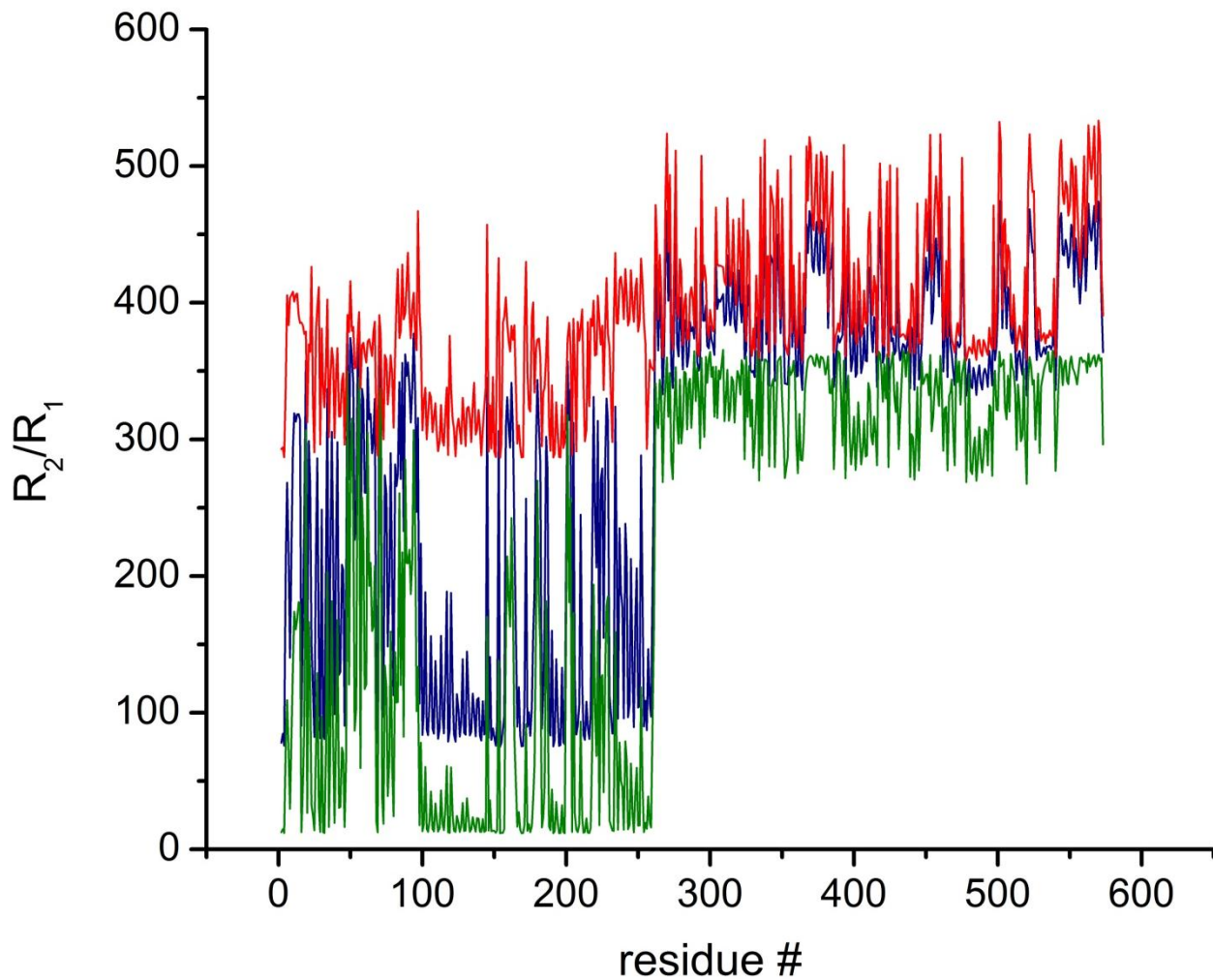


$$k_{slow} = 0.1 \times k$$

$$k = \frac{2}{\tau_D^A + \tau_D^B} = 155.57 \text{ [ns}^{-1}\text{]}$$

## Illustrative calculations

NMR relaxation rates for 600 MHz @ 300 K



$$k_{slow} = 0.1 \times k$$

$$k = \frac{2}{\tau_D^A + \tau_D^B} = 155.57 \text{ [ns}^{-1}\text{]}$$

$$k_{fast} = 10 \times k$$

## CONCLUSIONS

- Global restrains on overall molecular shape are useful for protein structure calculations. However, to achieve the best performances global restrains need to be supplemented by local restrains.
- Xplor-NIH is equipped with specific module for sampling molecular surfaces which opens the possibly to use the experimental restraints related to molecular surface topology for molecular structure computations.
- NMR relaxation data can be used as source of both global and local restrains simultaneously.
- Recent theoretical developments in combination with computational facilities of Xplor-NIH open the ways for new methods of simultaneous characterization of molecular structure and internal dynamics.

## ACKNOWLEDGEMENTS

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Amitabh Varshney

and YOU for your Attention